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13. ABSTRACT (Maximum 200 words)

This project was concerned with the problem of determining the physical and geometric properties of an unknown inhomogeneity from a knowledge of its effect on a given time-harmonic electromagnetic wave with particular concern to non-destructive testing and medical imaging. The main accomplishments were 1) The discovery of a linear method for determining the support of aberrant inhomogeneities without any a priori assumptions on either the frequency or magnitude of the inhomogeneity; 2) The application of this new linear method to problems in microwave medical imaging; 3) The analysis and numerical implementation of a method of "perfectly matched laye for the solution of Maxwell's equations; and 4) The derivation of an adaptive methoc for mesh refinement to produce a far field pattern of prescribed accuracy.

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Summary

This project was concerned with the problem of determining the physical and geometric properties of an unknown inhomogeneity from a knowledge of its effect on a given time-harmonic electromagnetic wave with particular concern to non-destructive testing and medical imaging. The main accomplishments were 1) The discovery of a linear method for determining the support of aberrant inhomogeneities without any a priori assumptions on either the frequency or magnitude of the inhomogeneity, 2) The application of this new linear method to problems in microwave medical imaging, 3) The analysis and numerical implementation of a method of "perfectly matched layers" for the solution of Maxwell's equations, and 4) The derivation of an adaptive method for mesh refinement to produce a far field pattern of prescribed accuracy.

Professional Personnel

Dr. David Colton	Unidel Professor of Mathematics University of Delaware Co-Principal Investigator
Dr. Peter Monk	Professor of Mathematics University of Delaware Co-Principal Investigator

Report on Research

The research of AFOSR Grant F49620-95-1-0067 was concerned with the development of new methods to solve the inverse scattering problem for fixed frequency time harmonic electromagnetic waves. The main problem under consideration was the determination of the support of aberreant inhomogeneities from a knowledge of the scattered wave. Such inverse problems are particularly relevant for anisotropic media where the inverse problem is strongly non-unique and determining the support of an anomaly is in general the best that can be expected. Fortunately, for many problems in medical imaging and non-destructive testing, such information is all that is needed.

In order to effectively study and analyze a proposed algorithm for solving an inverse scattering problem, it is necessary to have accurate near-field or far-field scattering data at one's disposal. This leads to the problem of developing reliable numerical schemes for solving the direct scattering problem for Maxwell's equations. A particularly attractive method for doing this was the idea of "perfectly matched layers" as originally proposed by Bérénger. The original ideas of Bérénger were extended and mathematically analysed by Collino and Monk [4] during the period of this research effort. Such an analysis was complicated by the non-standard nature of the modified Maxwell equations in the layer. Finally, given an efficient method for solving the direct scattering problem for Maxwell's equations, one wishes to produce a far-field or near-field radiation pattern of prescribed accuracy and to this end Monk and Suli [23] have investigated an adaptive method for mesh refinement to produce such radiation patterns.

Given accurate far-field (or near-field) data, one can now analyze inversion algorithms for inverse scattering problems. To this end, during the period of this research effort, we have developed a new method (called the "simple method") for determining the *support* of aberrant inhomogeneities rather than the precise *values* of the index of refraction [11][, [18]. In particular, the "simple method" mathematically places a grid on the material being probed, solves a *linear* integral equation of the first kind for each point on this grid, and then determines the support of the aberrant inhomogeneity from a knowledge of the solutions to the set of integral equations. Note that although the integral equations are linear, this is an exact method, i.e. no approximations have been made nor any assumptions on whether or not the frequency is high or low. The mathematical basis for this method rests

on showing the existence of a unique solution to a new class of boundary value problems for partial differential equations called interior transmission problems and the fact that this solution can be approximated by a Herglotz wave function.

In [16] and [17] we have used the "simple method" to study the problem of detecting tumors in the upper part of the lower leg and showed that the time for reconstruction was reduced from about three hours to fourteen seconds! Furthermore, it was shown in [13] that the "simple method" can also be used for certain types of anisotropic material, thus opening the door for the use of inverse scattering methods for materials of this type with particular reference to the study of the impact toughness of airplane canopies by non-destructive methods. The implications of this work for both medical imaging and non-destructive testing are obvious: if developed successfully this will be the first method for solving the inverse scattering problem that is rapid to implement but is independent of both ad hoc assumptions on the material and the frequency of the probing wave.

Personnel Supported

- Faculty
  D. Colton, P. Monk (Principal Investigators)

- Post-Doctoral Students
  R. Potthast (supported by Deutsche Forschungsgemeinshaft)
  M. Piana (supported by Consiglio Nazionale delle Ricerche)

- Graduate Students
  C. Labreuche (supported by Thomson CSF-LCR)
  Thesis: Inverse Obstacle Scattering Based on Resonant Frequencies (University of Paris)

  S. Fadoulourahmane (supported by Academy of Finland)
  Thesis: An Inverse Problem for Time Harmonic Electromagnetic Waves in an Inhomogeneous Orthotropic Medium (University of Oulu)

Publications

1. K. Chadan, D. Colton, L. Päivärinta and W. Rundell, *An Introduction to Inverse Scattering and Inverse Spectral Problems*, SIAM Monographs on Mathematical Modeling and Computation, SIAM, Philadelphia, 1997.

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Honors/Awards

Professor Colton was appointed Unidel Professor of Mathematical Sciences at the University of Delaware effective February 12, 1996.